used by Savart, the simple relation would have sufficed to define their behaviour.

(ix) The critical velocities, if they exist, obey a different law when, as in Savart's experiments, the liquid escapes through a sharp-edged hole in a horizontal plate. Our critical velocities owe their existence to the shape of the nozzle we have used. This favours turbulence—eddies or vortices—as indicated in the text.

*On Residual Magnetism in Relation to Magnetic Shielding.**

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(Communicated by Prof. J. A. Fleming, F.R.S. Received February 17, 1917.)

The present communication records the final stages of an investigation, of a somewhat exhaustive character, into the various problems which were presented by the necessity for constructing a magnetic shield suitable for large spaces, and capable of giving such a degree of shielding that the internal field caused by the earth should not exceed the order of magnitude 10^{-3} C.G.S. unit. The theoretical calculation of the best form of shield, and the details of its construction, were given, together with an examination of the various methods of testing the efficiencies of large shields, in the first paper.† A study of the effect of leakage through small air spaces was made at the same time, and it became apparent that not only this problem, but several others which are vital to the production of the theoretical efficiency of a shield, needed a more careful study than they had received hitherto.

The usual process adopted for removing permanent magnetism from a shell or set of shells, by the reversals of a slowly decreasing current, ceases to be efficient when the magnetisation is very small, unless special methods are introduced, and there was previously no definite indication of the degree of accuracy with which the magnetic induction at any point due to a coil, wound in a helix on one member of a set of spherical shells, could be either predicted or measured. A study of these problems was made, and the results described in a second paper.‡

* This investigation has again been facilitated by a grant made by the Council of the Society out of the Gore Fund.

† 'Roy. Soc. Proc.,' A, vol. 92, p. 529 (1916).

‡ 'Roy. Soc. Proc.,' A, vol. 93, p. 129 (1917).

It is now possible to correct the theoretical formulæ for the effect of the angle of the helix in a loose winding under these circumstances, and, by a method which adopts an exploring coil with only one turn, to measure accurately very small values of the magnetic induction, to an order as low as 0·7 C.G.S. unit, in any one of a set of spherical shells surrounded by one or more magnetising coils, with results in close accord with theoretical calculation. The ordinary process of demagnetisation of an iron shell is in this respect now on a definite—and, in fact, delicate—quantitative basis. In the same paper, moreover, it was shown that the delay in reversal of magnetic induction, caused by eddy currents, is effectively negligible when the magnetic induction is not greater than about 300 C.G.S. units. This fact removes what would otherwise be a source of considerable difficulty in the production of an effective shield, for a shield constructed according to the necessary theoretical specification, for large spaces, outlined in the first paper, is so constituted, as regards the thicknesses of individual shells and the distances between them, that currents which would ordinarily be used in removing permanent magnetism do not produce, in any individual shell, magnetic inductions which greatly exceed this value.

There is, however, one outstanding problem whose solution is necessary to complete the study of magnetic shielding on its experimental side. The necessity for a determination of what constitutes "effective demagnetisation" was pointed out in the second paper. In other words, there is a limit beyond which the permanent magnetism of a set of spherical shells cannot be removed by the use of magnetising coils in which currents are reversed and diminished. A study of this limit is one of the main objects of the work described in the present communication, which has shown that there is a phenomenon, of fundamental importance in the magnetic behaviour of iron, which has probably escaped attention on account of the previous restriction of accurate quantitative work on iron in large masses to the cases in which the magnetic induction is large. For one of the main conclusions of the present paper is that the permanent magnetism of a mass of iron can only be removed by the reversal and diminution of a current when all *steady* fields, of which that of the earth is a particular example, are absent. Although the field of the earth is small, it is, nevertheless, very large in comparison with the limiting field which is desired in the interior of the magnetic shield, and the succeeding experiments show that, when it is not approximately neutralised by a current flowing in a coil just outside the shield, the reversals of current in any other coil on an inner shell are, in certain cases, capable of producing a stronger degree of polarisation than they were actually designed to remove. The whole shield must, in fact, be surrounded,

during the process of demagnetisation, by a current which will reduce the field just outside the shield as nearly as possible to zero.

The details of the apparatus have been described sufficiently in the previous papers. We may at this point merely recall that the shield consists of four concentric spherical shells, numbered 1, 2, 3, 4, from innermost to outermost, and wound with magnetising coils, to which the same numbers relate. Although each shell was originally a pair of hemispheres in contact, the junction planes have now been machined so that each half of any one shell penetrates the other half, the method being shown in fig. 1. Except in the case of the second shell, there is a small

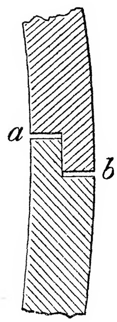
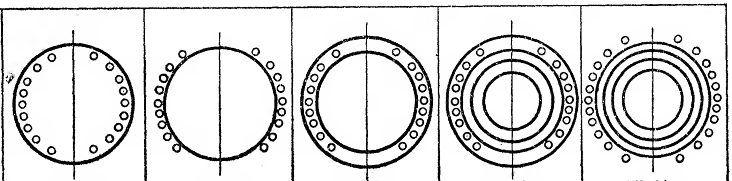


FIG. 1



Number of shell	Units	Units	Units	Units	Units
1				−0·45	−0·06
2				−1·38	−0·17
3	+1	−2	−6·38	−4·14	−0·42
4			+4·71	+5·52	−1·26

FIG. 2.

remaining air space of the form indicated in the figure, but in other respects the closeness of fit is quite satisfactory. It is, in fact, the closeness of fit which prevents complete contact at the points *a*, *b*, and it will be noticed that there is no continuous air space extending through the material of the shell at any point.

The junction planes of the shells are all placed at right angles to the direction of the earth's field, and the magnetising coils are normal to the diameter in the direction of this field. The axis of the inductor ring inside the shells is at right angles to the magnetic meridian, and coincides with a diameter in the planes of junction. These adjustments are identical with those made in the previous experiments, and the work has again been carried on in the Siemens Laboratory at King's College.

Throughout this communication, forces are called positive when they have the same sign as that of the earth, and positive forces are plotted above the axis of the abscissæ in all the diagrams. The subsequent curves show directly the relation between the average magnetic intensity in the hollow of the shield and the angular position of the inductor ring. The plane of the latter is vertical when the angle is 0°, and horizontal when it is 90°. The

direction of the earth's magnetic field corresponds to an angle 67° on the scale of abscissæ. Further details may be found in earlier papers.*

Preliminary experiments with the smallest shell showed that the machining had greatly improved its efficiency as a shield, by the considerable reduction in magnetic leakage. In the original experiments of the first paper, before the shell was machined, the field inside it was 0·116 C.G.S. unit. This is now reduced to 0·06 C.G.S. unit without the use of appreciable demagnetisation of its material, and the curve obtained by rotating the inductor ring within it is much more nearly a sine function of angle; this is an indication of diminished leakage and of uniformity in the internal field. Experiments with groups of two and of three shells indicated a similar degree of improvement, but the details of this work are not given, for the main object in view was the investigation of the ultimate shielding ratio which can be attained in practice with the four complete shells arranged with a common centre, and all the characteristic features of the curves obtained with a selection from the shells are reproduced in the experiments involving four shells, in which, on account of the small forces then concerned, measurements of a much greater delicacy can be made, and disturbing factors are of greater relative importance.

Experiments with Four Complete Shells.

Before proceeding to an account of these experiments, it is convenient to collect together, for reference, certain data which were calculated in the last paper,† together with some further results of calculation found by the same method, which can be verified readily by the reader. They relate, of course, only to the particular set of shells used throughout these experiments, but in their general scope they give valuable information in connection with any magnetic shield built up according to a specification of maximum efficiency. In the estimation of the maximum value of the magnetic induction for various combinations produced by a definite current, it is convenient to take a single shell wound with a coil carrying that current as a unit. This is done in the diagram (fig. 2), where the third shell alone is the unit. The magnetising coil used in any other case is indicated in the figure by a set of small circles. The large circles represent the shells forming part of any configuration. The coil on the third shell as unit was wound on the inner surface.

With this figure may be combined the statement that a current of C ampères in the magnetising coil produces a maximum magnetic induction

* 'Roy. Soc. Proc.,' A, vol. 92, p. 529 (1916).
† 'Roy. Soc. Proc.,' A, vol. 93, p. 129 (1917).

of 5·9 C.G.S. units in the single shell taken as standard. The figure is therefore such that 1 unit = 5·9 C units of magnetic induction. This value, which relates to small currents, must be increased for larger ones owing to the influence of the curve of magnetisation.

The experiments described in the preceding paper left the shells in a polarised condition as the result of the last applied current of 5·5 ampères in the third magnetising coil. This current gives rise to maximum magnetic induction of amounts $B_{max.} = -20, -68, -204, +272$ C.G.S. units in the shells numbered 1, 2, 3, 4 respectively, where the first value is approximate. From this starting point, the first new experiments made consisted of the connection of the magnetising coils 1, 2, 3 in series, in order to give alternating signs to their effects on any particular shell—this question of sign was considered in the second paper—and the passage of a current of 4·7 ampères through them initially. This current was gradually reduced by stages to 0·01 ampère. Ten reversals were made during each stage, the time interval between each reversal being four seconds, and the number of such stages or steps in the reduction was 40. The full line curve *o* in fig. 3 exhibits

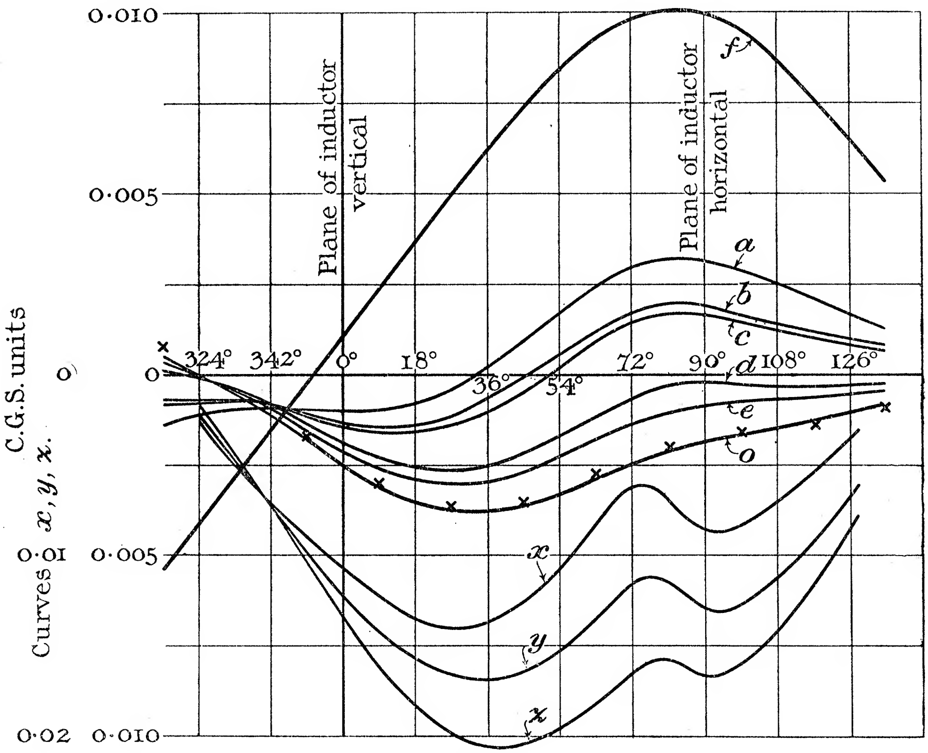


FIG. 3.

the result, and it demonstrates the final existence of a field of 0·0038 C.G.S. unit *opposed* to that of the earth, in the space internal to the four shells.

It is desirable at once to place emphasis on this important experimental result, for it has been obtained consistently throughout the work with these spherical shells. As an illustration, we may refer back to the first paper,* in which, the shells being then in their initial state before machining, the same character was obtained in the results. For the treatment to which they were subjected, prior to the determination of the curves in fig. 3 of that paper, was demagnetisation by the use of coils 1, 2, 3, in this order, but separately, the initial maximum currents being 7, 8, 7·5 ampères respectively. The curves x, y, z in fig. 3 of the present paper were then obtained by rotation of the inductor, and will be referred to again in connection with later experiments which bear out the same conclusion.

It might be thought that the poles or intersections of any shell with the diameter which is axial to all the turns of a magnetising coil are connected with this effect. For, in their neighbourhood, the magnetic induction produced by a current in the magnetising coil is notably smaller, and effective demagnetisation of a shell in these regions would be difficult. In order to test this point, another coil was wound on the third shell at right angles to the coil already present, and embracing an angle of about 10° on either side of the axis of this coil, thus covering an appreciable zone of the shell at the poles. It had six turns, and its axis, normal to its plane of winding, coincided with the axis of the inductor. A current of 6·8 ampères was passed through this new coil and gradually reduced during reversal by steps to the value 1/13000 ampère. The only effect produced was a slight change of phase of the curve o, fig. 3, with no change in amplitude Demagnetisation by the use of coils 1, 2, 3 from the previous terminal value of 0·01 ampère to 0·00006 ampère was then carried out without producing any effect, on the internal field in the shell, which could be measured. It is therefore evident that we are in the presence of a phenomenon which does not arise from the poles of the coils. Up to this point no current had been applied to the fourth or outermost coil in the present experiments.

The internal force of 0·0038 C.G.S. unit, which was now present, could not, from its sign, represent a partially shielded field of the earth, but necessarily had its origin in some disturbing factor which could only be regarded as residual magnetisation incapable of removal by reversed and diminished currents of moderate amounts in the three inner magnetising coils. As a preliminary experiment, designed to elucidate the nature of the effect, a current of 0·457 ampère was passed through the fourth or outermost

* 'Roy. Soc. Proc.,' A. vol. 92, p. 529 (1916).

coil in such a direction as to oppose the earth's field in the interior of the coil, and maintained constant. During the passage of this steady current, the process of demagnetisation was carried out in the usual manner by passing a current of 4·7 ampères through coils 1, 2, 3 arranged in series, and reducing it by steps, with many reversals, to 1/13000 ampère. The circuit in the fourth coil was then broken, and curve *a*, fig. 3, taken during rotation of the inductor. This curve indicates a positive maximum force, of the same sign as the earth's field, in the interior of the shield. The actual value of this force is +0·00324 C.G.S. unit. It is evident that this will include an effect due to permanent polarisation mainly of the fourth shell, round which a steady current was maintained.

This current of 0·457 ampère round the fourth shell was then restored, and gradually reduced by steps during reversal to 1/6000 ampère. Curve *b*, fig. 3, was then obtained by the examination of the internal space. The maximum force in the space, as read from curve *b*, is +0·00193 C.G.S. unit, notably smaller, but again of the same sign as the earth's field. The shells have apparently at this stage been demagnetised very completely throughout the whole of their mass, but in order to ensure complete treatment, currents of 0·01, 0·109, 0·555 ampère were passed successively through coils 1, 2, 3 in series, and reduced in each case by small steps with many reversals to 1/6000 ampère, so that all the shells should now be demagnetised down to this value. The use of the current 0·01 ampère produced no difference capable of detection in the internal force. The currents 0·109 and 0·555 produced some differences, and the results of their application are shown in curves *c* and *d* respectively. Demagnetisation with the large current, 0·555 ampère, was repeated, but it only led to a very slight reduction in the force shown by curve *d*. A larger initial current of 1·05 ampère was then used in place of 0·555 ampère, and the process repeated. Curve *e* shows the result. Finally, the same process was carried out with an initial current of 2·5 ampères, and the original curve *o* was restored, the observations made during the rotation of the inductor in this experiment being indicated in the figure by crosses on the original curve *o*.

In this manner, it was therefore found possible to remove the negative force of 0·0038 C.G.S. unit by the actual establishment of residual magnetism, and subsequently to restore it again by demagnetisation. At the same time a considerable degree of control was established over the value of the internal force within the shield. It will be noticed that the minimum field obtained up to this point is given by curve *c*, and has the value 0·00175 C.G.S. unit approximately in the direction of the earth's field, and with it is a negative field of 0·0016 unit associated primarily with leakage.

This result, which is the best obtained hitherto from the point of view of close shielding, can, however, be improved. The essential feature to which attention should be drawn is the use of the original steady current of 0·457 ampère in the outermost coil. For this constituted the only difference in procedure adopted in this set of experiments when compared with those made previously, and the more satisfactory results obtained must in some manner be due to it. This conclusion is enhanced by the fact of the restoration of the original curve *o* when a demagnetisation of the shells was carried out, without the presence of this steady current in the fourth coil, with a current exceeding 2 ampères initially.

In order to illustrate and examine this process further, a larger current of 0·893 ampère was then passed through the fourth coil and maintained constant during the demagnetisation of the shells from an initial current of 5 ampères in coils 1, 2, 3 arranged in series; the current of 0·893 being finally reduced by steps during reversal to 1/6000 ampère. Curve *f* was then obtained for the internal space, and has a maximum ordinate of +0·0101 C.G.S unit, whereas the maximum ordinate prior to demagnetisation with the fourth coil had the slightly greater value +0·0122 unit.

Fig. 4 shows the effect of carrying this process to a further extent. A current as large as 2·5 ampères was passed through the fourth coil in such a direction as to oppose the earth's field in its interior, and maintained constant. Meanwhile the shells were demagnetised by a current of initial value 9·2 ampères passed through the coils 1, 2, 3 in series, and reduced during reversals to 1/6000 ampère. After breaking the current in the fourth coil, curve 1, in fig. 4, was obtained as the representation of the inside field. Its maximum ordinate is +0·0275 C.G.S. unit, and on demagnetising subsequently with the fourth coil from 2·4 ampères downwards, this ordinate was reduced to +0·0224 unit, as shown on curve 2 of the same figure. The remaining processes employed in the experiments which led to fig. 3 were then carried out for fig. 4. Thus a series of demagnetisations was made with initial currents of ascending values 4·8, 7·5, 10·2, 12·5 ampères in coils 1, 2, 3, arranged in series as before, and the maximum ordinates corresponding to these operations were +0·0109, +0·0052, +0·0042, and −0·008 C.G.S. unit respectively, as shown on curves 3, 4, 5, 6 of fig. 4. As before, by the application of a sufficiently large demagnetising current (12·5 ampères), the positive force within the shield, given by curve 1, has been removed and a negative force obtained. It will be noticed that curve 6 has an ordinate of magnitude 0·0073 C.G.S. unit at 78° in a direction parallel but of opposite sign to the earth's field, and that the ordinates on either side at angles 51° and 102°, which are equidistant from the position,

are $-0{\cdot}008$ and $-0{\cdot}00795$ unit respectively, that is to say, are effectively equal.

On comparison of curve 6 (fig. 4) with curves x, y, z in fig. 3, it will be

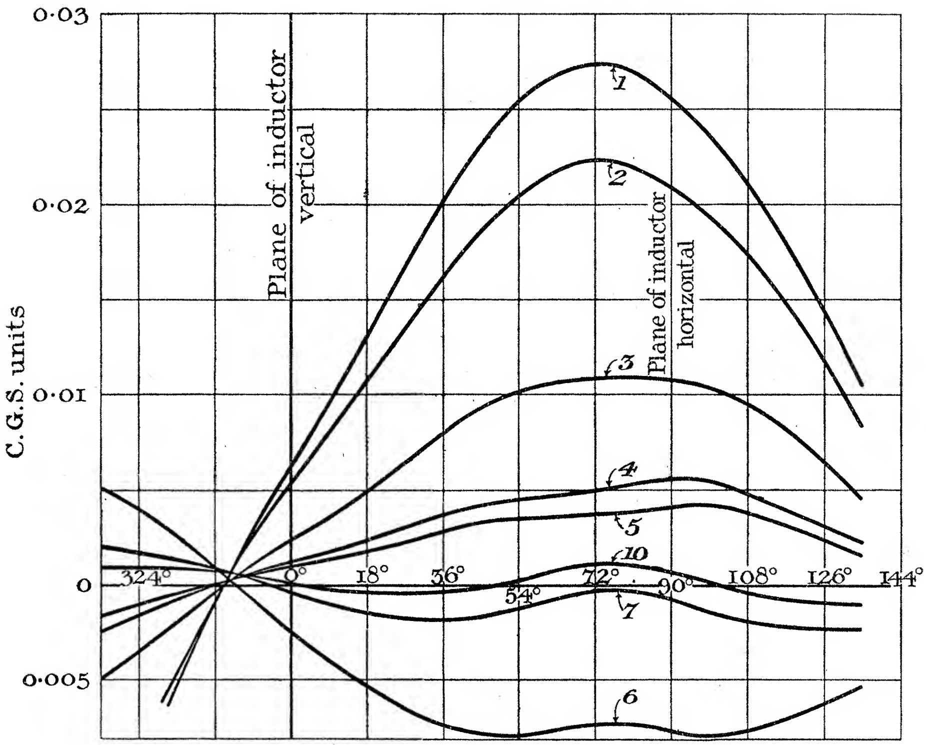


Fig. 4.

seen that they all have two points of inflexion, whereas curve o (fig. 3) has no such points. Experience with these shells has demonstrated the necessity for removal of every trace of polarisation, until the ultimate negative force in the shield gives this particular wave form when the inductor is rotated. If this wave form has once been obtained, it is possible to proceed to the highest degree of shielding. We have shown already that the best shielding result obtained with curve o (fig. 3) as a basis, involved a final internal force of $+0{\cdot}00175$ C.G.S. unit, associated with a leakage field of $0{\cdot}0016$ unit, approximately. It was stated at the time that the results could be improved, and it is, in fact, shown later that a force as low as $+0{\cdot}00117$ C.G.S. unit can be obtained. The importance of the points of inflexion is contained in the fact that their existence is a general indication of the approximate absence of polarisation, capable of removal by the magnetising coils, the wave form then denoting a mere superposition of the shielded

effect of the earth, which is a uniform field, giving a pure sine curve when the inductor is rotated, and the leakage field with a more restricted degree of symmetry. When these points of inflexion are once obtained, they persist in a remarkable manner in the subsequent curves derived by variation of the demagnetising currents in the four coils. For example, reference to the earlier paper will show that the curves 1 and 2 (fig. 3) of that paper,* which were obtained after the curves x, y, z in fig. 3 of the present paper, maintain the same characteristic wave form.

In the further development of the work, the curves are plotted on a larger scale in fig. 5. A current of 0·675 ampère was now maintained constantly in the fourth coil in such a direction as to oppose the field of the

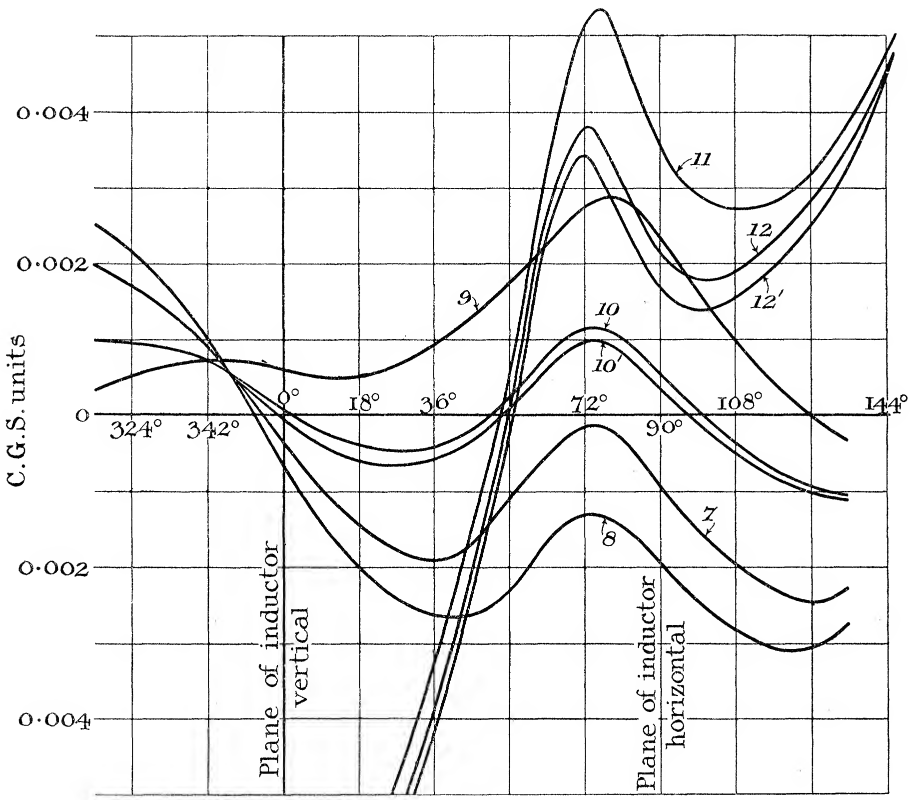


Fig. 5.

earth inside the coil. The process of demagnetisation of the shells was carried out simultaneously from an initial current of 1·9 ampères in the coils 1, 2, 3, arranged in series. The current in the fourth coil was then broken, and the interior of the shield explored by the inductor ring, with

* 'Roy. Soc. Proc.,' A, vol. 92, p. 546 (1916).

the result shown in curve 7 (fig. 5). The current in the fourth coil was then restored and reduced by steps during reversals. Curve 8 (fig. 5) was then taken, and it indicates a negative force directly opposed to the earth's field, of magnitude $-0{\cdot}0013$ C.G.S. unit.

The current in the outer coil was then maintained at 0·751 ampère, and the shells demagnetised as before from 1·9 ampères. The internal field obtained on breaking the outer current is shown in curve 9 (fig. 5). The subsequent demagnetisation by the fourth coil from 0·751 ampère changes the distribution into that indicated in curve 10 (fig. 5). This curve has ordinates at angles 315° and 135°, equal to one another, and of magnitude 0·001 C.G.S. unit, and the ordinate at 72°, coinciding with the direction of the earth's field, indicates a positive force of 0·00117 C.G.S. unit. The coincidence at about 70°, and not at a more displaced position as, for example, in fig. 3, indicates that the process is complete. It is therefore evident that, in order to obtain a maximum value of the shielding ratio, the shells must in the first instance be demagnetised very closely, with an initial current in the coils 1, 2, 3, even so large as 12 ampères, the maximum magnetic inductions produced by this current in the shells being relatively large. Even larger currents might be required if the polarisation in the shells were more important. The force 0·00117 C.G.S. unit at the angle 72° is regarded, in accordance with the argument in the first paper, as the true effect, within the internal space, of the earth's shielded field.

After the iron had been left for 21 days in the condition produced by the experiments which led to curve 10 (fig. 5), another exploration of the interior of the shield was conducted by rotating the inductor ring. In this manner the effect of resting of the material can be tested, and the result of the new experiment is shown in curve 10′ (fig. 5). The change due to a considerable rest is seen by comparison of curves 10 and 10′ to be very slight, and it is in a direction indicating that the field of the earth is tending to regain its original polarising effect. At the same time, the almost exact parallelism of the curves is an indication of the high degree of accuracy with which the inner space can be explored by an inductor adjusted in this manner.

It is interesting to notice that the current 0·751 ampère, which was used finally in the fourth coil in such a direction as to oppose the earth's field within the coil, is identical with the current used in the earlier paper, and, as was pointed out at the time, it is the current which just suffices to annul the force due to the earth* in the hollow of the shield. The experiments, in fact, lead to the conclusion, stated in the introductory remarks in this

* 'Roy. Soc. Proc.,' A, vol. 92, p. 545 (1916).

communication, that an essential feature of the effective demagnetisation of the shells is the use of an enclosing current which will annul the direct action of the earth, so that reversal continually takes place in the whole magnetic induction at every point. Iron behaves in such a manner that a steady field and an alternating and diminishing field applied together leave a joint effect of permanent magnetism in the material. This conclusion has been obtained before* in the case in which the steady field is due to a coil carrying current. It must now be extended to a small permanent field such as that of the earth, and the present effect is merely a manifestation of the same phenomenon on a smaller scale, but with important consequences on account of the very small magnetic effects which must be measured in the present experiments. The fact that it has not been noticed by earlier workers is probably due solely to the more limited degree of accuracy with which the larger fields employed by them could be, or needed to be, measured.

The importance of leakage due to the small air-spaces at the junctions of the shells, even after machining, requires some emphasis in so far as it affects the shielding produced. If it were entirely absent, the field in the interior, however small, should be uniform when polarisation of an irregular type is removed; and it should therefore be possible by the aid of residual magnetism, produced by a current of the proper amount applied to a magnetising coil, to so polarise the shells that the resultant field in the interior should be absolutely zero. But the leakage field is not uniform, and must, in fact, from obvious theoretical considerations be of some complexity, so that no simple arrangement of coils carrying currents can be devised which will, for internal points, completely annul this field. The experiments have shown that the machining of the shells has very greatly reduced the leakage, which is now of order 0·001 C.G.S. unit, instead of the original 0·006 unit. A portion of the original direct field, parallel to that of the earth, is, of course, due also to leakage, and this fact explains some of the reduction of this field from 0·0038 to 0·00117 unit.

For the purpose of comparison with the results obtained before the shells were machined, the curves 11, 12, 12′ of fig. 5 are reproduced from the earlier paper. Curve 11 was obtained after the shells had been demagnetised from an initial current of 3 ampères in coils 1, 2, 3 in series, during such time as a current of 0·746 ampère was maintained constant in coil 4, and in opposition to the earth's field in the interior of the coil. Curve 11 was determined after the breaking of this steady current, and curve 12 after the current had been restored in the fourth coil, and reduced after the usual

* 'Roy. Soc. Proc.,' A, vol. 83, p. 1 (1909).

manner to zero. Curve 12′ was observed under the same circumstances as curve 12, after an interval of four days, and shows the effect of a period of rest. The relation between 12 and 12′ is identical with that between 10 and 10′ described already.

It is, perhaps, unnecessary to remark at length upon the manner in which the more curious features of the curves obtained in the first paper have now received a satisfactory interpretation. In the testing of the shells as received originally from the foundry, the reversal of the internal force which was obtained was attributed to a possible polarisation of the shells produced by the earth's field during the process of cooling. In the light of the present experiments, it is apparent that the effect was, if not wholly, at least mainly, due to polarisation produced at the time of test. For the amplitude of the reversed internal force is 0·0038 C.G.S. unit when the initial demagnetising current in coils 1, 2, 3 is 4·7 ampères, and it is 0·008 due to demagnetisation from 12·5 ampères. In the latter case, the shells were highly polarised before demagnetisation. In the experiments of the earlier paper, before machining of the shells, a force of value 0·02 was observed after demagnetisation, as shown by curves x, y, z, in fig. 3 of the present paper, and this is in substantial agreement with the force observed when the shells were tested as received from the foundry. A more detailed examination does not appear to be required.

It is essential to success that the iron shall be demagnetised when free from the direct action of the earth's field. This approximate freedom from direct action can be secured sufficiently in practice by a somewhat wide range of current in the outer coil.

It is well known that if iron is subjected to a considerable magnetising force and then tested for permeability at a lower force, the permeability obtained is less than it would have been in the event of previous demagnetisation of the iron. The present experiments show that the earth's magnetic field produces a polarisation which by analogy should imply a higher permeability when removed. The increase of permeability which occurs in a specimen of iron demagnetised in a magnetic shield thus receives an interpretation in the light of these results.

Some Effects Due to Residual Magnetism.

In the following experiments, currents of gradually increasing amount were passed through the fourth magnetising coil, in each case (a) positively, in order to assist the field of the earth within the shield, and (b) negatively, in order to oppose it. The force in the interior of the shield was determined by the inductor as a wave form in four cases:

(1) when any given current was flowing positively, (2) when that current was broken, (3) when the same current was flowing negatively, and (4) when this current was broken. These operations were performed for a sequence of currents. Fig. 6 exhibits a sequence of wave forms obtained

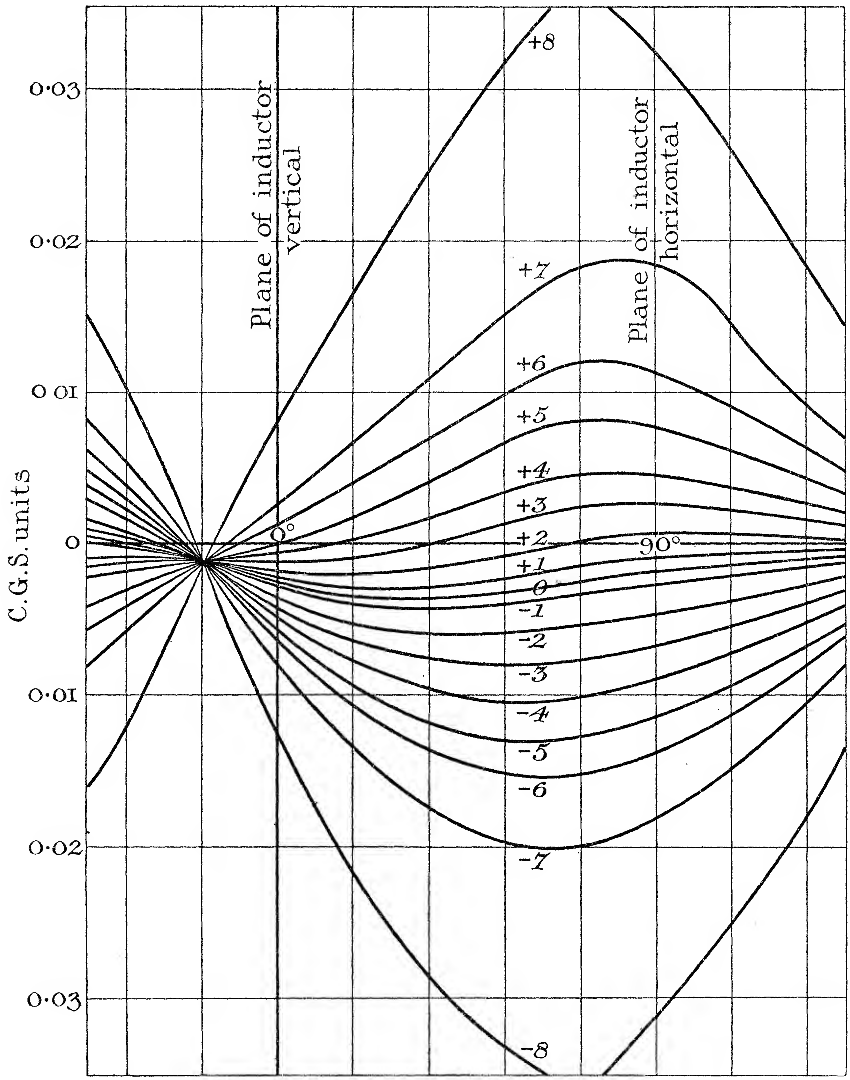


FIG. 6.

for the internal space. Those corresponding to the various positive currents are curves $+1$, $+2$, $+3, \ldots$, whereas curves -1, -2, $-3, \ldots$, relate to the corresponding equal negative currents. The values of the currents in ampères are given in the accompanying Table (Table I). The internal force

Table I.—(Wave forms read downwards).

Ampères in fourth coil		0.	0·0708.	0·191.	0·365.	0·583.	0·822.	1·15.	1·73.	3·95.
Angle.	Curve number, fig. 6, + or –.	0.	1.	2.	3.	4.	5.	6.	7.	8.
	After application of—									
– 45°	Positive current	—	120	120	126	130	123	132	108	74
	Negative current	+103	121	116	121	112	106	109	126	155
	Difference	(+17)	– 1	+ 4	+ 5	+ 18	+ 17	+ 23	– 18	– 81
– 27°	+	—	72	86	88	91	92	81	76	99
	–	– 91	77	76	84	86	100	90	79	83
	Difference	(–19)	– 5	+ 10	+ 4	+ 5	– 8	– 9	– 3	+ 16
– 9°	+	—	359	356	354	374	370	371	356	348
	–	–358	350	352	354	359	377	369	363	359
	Difference	(+1)	+ 9	+ 4	0	+ 15	– 7	+ 2	– 7	– 11
9°	+	—	629	620	621	639	651	660	619	576
	–	–627	621	614	617	624	643	642	640	654
	Difference	(+2)	+ 8	+ 6	+ 4	+ 15	+ 8	+ 18	– 21	– 78
27°	+	—	722	724	721	741	750	768	713	652
	–	–736	718	722	714	718	737	746	749	767
	Difference	(–14)	+ 4	+ 2	+ 7	+ 23	+ 13	+ 22	– 36	–115
45°	+	—	693	699	690	705	722	752	682	588
	–	–724	697	690	675	682	708	706	718	740
	Difference	(–31)	– 4	+ 9	+ 15	+ 23	+ 14	+ 46	– 36	–152
63°	+	—	542	543	549	550	567	597	515	436
	–	–568	549	542	532	521	535	560	603	587
	Difference	(–26)	– 7	+ 1	+ 17	+ 29	+ 32	+ 37	– 88	–151
81°	+	—	397	401	402	403	414	445	370	281
	–	–426	405	404	385	381	394	407	406	449
	Difference	(–29)	– 8	– 3	+ 17	+ 22	+ 20	+ 38	– 36	–168
99°	+	—	320	320	307	317	315	337	280	213
	–	–333	319	321	305	291	302	315	322	357
	Difference	(–13)	+ 1	– 1	+ 2	+ 26	+ 13	+ 22	– 42	–144
117°	+	—	255	258	246	244	242	258	213	160
	–	–260	255	248	241	258	228	255	250	285
	Difference	(–5)	0	+ 10	+ 5	– 14	+ 14	+ 3	– 37	–125
135°	+	—	161	157	151	144	135	144	116	87
	–	–161	159	157	152	130	125	136	146	180
	Difference	(0)	+ 2	0	– 1	+ 14	+ 10	+ 8	– 30	– 93

before these operations were commenced was that shown by curve *o* (fig. 3), which is reproduced in fig. 6, on a smaller scale, and again called curve *o* in the new figure. The permanent effect of these successive positive and negative magnetisations upon the curve *o* has been small in comparison with the magnitudes of the corresponding forces during the intervals of time in which the various currents were flowing. It is, however, not possible to exhibit on a small scale with clearness all the curves showing the permanent alterations in the internal force due to the successive temporary positive and negative magnetisations. But the ordinates of the resulting curves are shown in the Table in divisions of the galvanometer scale, and one such division represents $5{\cdot}06 \times 10^{-6}$ C.G.S. unit. The differences shown in brackets in the Table indicate the initial differences produced by the smallest applied current which was adopted. Differences without brackets are the effects of the reversal of the current. The first current applied in the case of each wave form had a positive sign.

The Effects of Hysteresis.

Fig. 6 and Table I illustrate some of the effects of hysteresis, which is, of course, the agency producing the residual magnetism introduced into the shells at the time of test. If a current be passed through the fourth coil and then broken, the hysteresis in the iron of each shell will cause, in varying degree, a certain residual magnetic induction which must produce an internal magnetic force superposed on the force already existing. In these experiments, this new force is negative, as indicated by curve *o* (fig. 6). After the application of the positive currents 1·73 and 3·95 ampères, corresponding to curves $+7$ and $+8$ of fig. 6, the internal force is diminished, showing that the new residual magnetism has produced a positive force. With currents of this magnitude, the polarisation begins to be strong. On the other hand, currents of magnitudes 0·583, 0·822, 1·15 ampères, as shown by curves $+4$, $+5$, $+6$, produce a contrary effect. Possible errors of observation make the differences in the case of currents 0·0708, 0·191 ampère, as in curves $+2$ and $+3$, less reliable, but, at the same time, the differences obtained have a pronounced positive tendency on the whole, and the effect cannot well be mistaken. The general conclusion derived from the Table can be summarised in the statement that for currents ranging between the values 0·365 and 1·15 ampères, there is a reversal in sign of the residual effects. The mean of these values is 0·75 ampère, which is the current most effective in neutralising the earth's field in the interior of the shells in the preceding set of experiments. The range of currents is equal on either side of this value, and over the whole range there is, when the current has a negative sign,

a rough approximate neutralisation of the earth's field. It is suggested, therefore, that this reversal in the sign of the residual effects is connected with the fact that the earth's field was operative during the present set of experiments, and that the reversal is, in fact, due to the presence of this field. This throws light upon the nature of the action of a steady field in preventing complete demagnetisation of a mass of iron.

Tests for hysteresis loss have recently been made in the case of the circular transformer iron stampings originally used as a magnetic shield by one of us,* and the results are given in Table II. They are of special interest in this connection because the initial permeability of the material approximates very closely to that of the present spherical shells. It should be noticed that these stampings had not been magnetised previously except by the magnetic field of the earth, and that the specimen was exposed to this field during the test. The total volume of iron used was 23,440 c.c., and the average thickness of the stampings was 0·0613 cm. Each had internal and external diameters of 30·5 and 40·6 cm., and the cross-sectional area of the cylinder into which they were formed was 210 sq. cm.

In fig. 7, the residual magnetism B_0 of this specimen is plotted in terms of the

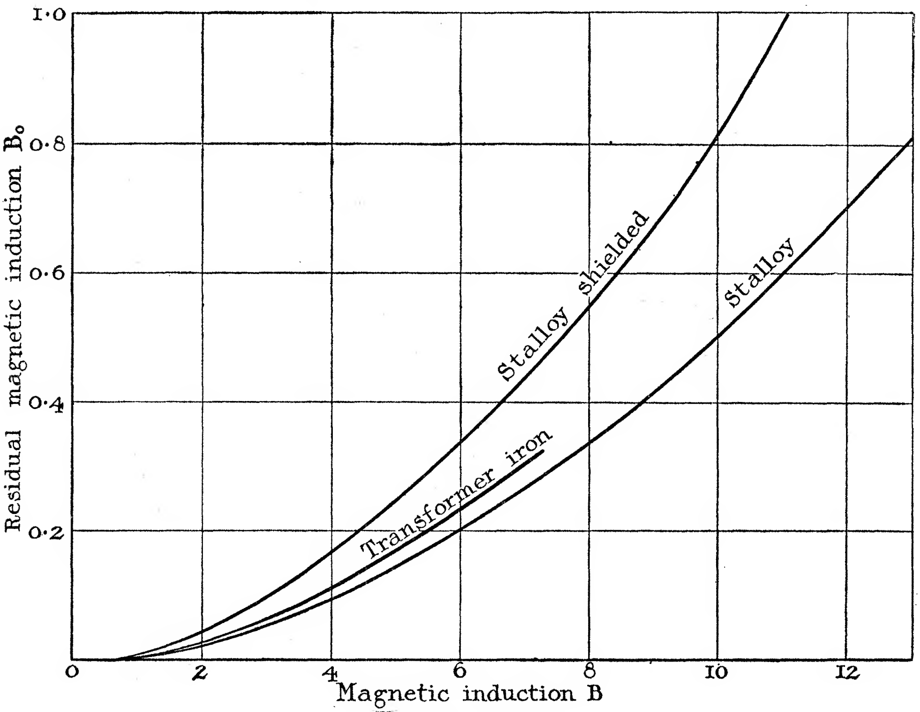


Fig. 7.

* 'Roy. Soc. Proc.,' A, vol. 90, p. 179 (1914).

value of $B_{max.}$ present during its production. For the purpose of comparison, the corresponding curves for a specimen of "Stalloy," or silicon-iron alloy, are shown in the same figure before and after treatment with the earlier form of shield described some time ago.*

Table II.

	Maximum H.	Maximum B.	Permeability.	Residual magnetism B_0.	Coercive force, H_0 (in 10^{-6}).	Ergs per cycle per c.c. (in 10^{-6}).
Transformer iron	0·0000734	0·0109	148			
	0·0001147	0·0170	148			
	0·000234	0·0354	151			
	0·000437	0·0632	150			
	0·000806	0·123	153	0·00070	4·6	0·16
	0·00131	0·203	155	0·00076	4·95	0·31
	0·00194	0·302	156	0·00092	5·84	0·56
	0·00348	0·542	156	0·00206	13·2	2·28
	0·00724	1·15	159	0·0074	56	15·8
	0·0143	2·36	165	0·040	280	152
	0·0214	3·59	167	0·085	510	514
	0·0356	6·13	172	0·244	1,500	2,150
Stalloy before treatment in magnetic shield	0·00168	0·462	275			
	0·00420	1·18	281	0·016	47	17·5
	0·00843	2·44	289			
	0·0179	5·55	310	0·18	500	861
	0·0496	19·6	395	1·47	4,000	18,100
	0·0935	45·7	489	4·45	10,000	102,000
Stalloy after treatment in magnetic shield	0·00168	0·68	405	0·0068	16	3·1
	0·00420	1·72	410	0·028	58	30·6
	0·00843	3·59	426	0·13	310	279
	0·0179	8·92	498	0·65	1,400	2,910
	0·0496	36·4	734	4·9	6,300	58,300
	0·0935	88·3	944	14·0	13,800	306,000

On the Effect of Sliding Brush Contacts.

When measuring the strength of an internal field of the order 0·002 C.G.S. unit, it was possible to cut out the extra resistance in the galvanometer and inductor circuit, and to obtain a deflection within the limits of the scale, and of such magnitude that the permanent set in the suspension fibre due to twist was still negligible. It was also possible to obtain readings of considerable accuracy when a resistance, so large as 10,000 ohms was added to the circuit. In fig. 8 the curves 1, 2, 3 were obtained with 10,000 ohms, 2000 ohms, and no added resistance respectively, and it will be noticed that the amplitude of the maximum ordinate at angle 54° is variable. Moreover, the ordinates at the angles 0° and 180° are not equal, as they should be,

* 'Roy. Soc. Proc.,' A, vol. 90, p. 179 (1914).

since we are dealing with a function which is periodic in regard to each half period.

If the ordinates at 0° and 180° are made equal in the case of each curve

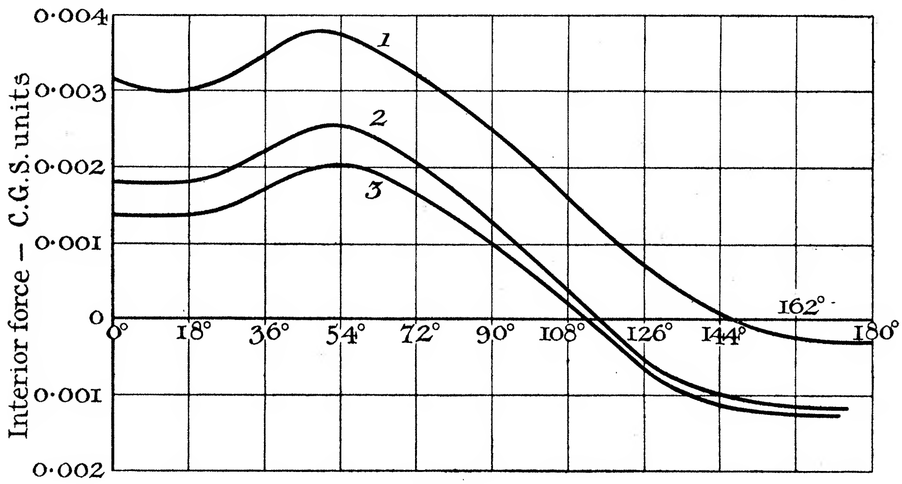


Fig. 8.

by raising the horizontal axis, then the maximum ordinate is found to vary as shown in the following Table.

Table III.

Added resistance in ohms.	Maximum ordinate at 54°.				
	1.	2.	3.	4.	5.
10,000	0 ·00203	0 ·00228	0 ·00205	—	—
2,000	0 ·00208	0 ·00223	—	0 ·00214	0 ·00216
0	0 ·00181	0 ·00195	0 ·00192	0 ·00192	0 ·00184

Comparison of the results obtained in any one experiment shows that the force is consistently smaller when no resistance is added. The forces with 10,000 and 2000 ohms additional resistance are substantially equal. Since the force was actually constant throughout any one experiment, we conclude that the displacement of the horizontal axis is connected in some way with the brush contact, and is a function of the current passing through the junction. The difference between the ordinates with no added resistance and either 10,000 or 2000 ohms added, after making the adjustment of the horizontal axis, is attributed to the damping of the instrument during the time of contact of the surfaces. This time depends upon the operator, and

varies from $\frac{1}{4}$ to $\frac{1}{3}$ second. The inductor and its sliding contacts are described in an earlier paper.*

Summary.

1. The paper contains a further contribution to the study of the problems presented by the necessity for constructing a magnetic shield capable of reducing the earth's field to an order as low as 0·001 C.G.S. unit in a large space. The main problem, not treated in earlier papers, is that of residual magnetism in the various shells of the shield, and this problem is discussed in connection with exhaustive experiments in the present paper.

2. It is found that the ordinary process of demagnetisation of a mass of iron fails to be completely effective if, during the operation of the current, which is diminished by steps and continually reversed, a constant magnetic field such as that of the earth is present at the same time. This phenomenon has escaped notice hitherto, probably on account of the smallness of the earth's field, but it becomes prominent in experimental work involving the measurement of fields so small as that specified in (1).

3. This effect of the steady magnetic field is shown to be associated with a reversal of the residual effects of hysteresis in iron when tested in the earth's field by currents lying within a certain range in which they approximately annul the field.

4. It has been found possible to ensure complete removal of irregular polarisation or previous magnetic history of the shells, provided that during the preliminary demagnetisation of the shells the earth's steady field on them is annulled by a steady current of suitable amount enclosing the whole shield.

5. The magnetic shield described in earlier papers has now the degree of shielding which is required by theory in view of the known values of the permeabilities of its component shells. Its properties do not change appreciably with lapse of time.

6. The well known fact that iron polarised by a large force, and subsequently tested for permeability at a lower force, shows diminished permeability at the lower force gives, in combination with these results, an interpretation of the increase of permeability manifested by iron when tested within a magnetic shield.

* 'Roy. Soc. Proc.,' A, vol. 92, p. 540 (1916).